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Reference Command Tracking for a Linearized Model of an Air-breathing Hypersonic Vehicle

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The focus of this paper is on control design and simulation for an air-breathing hypersonic vehicle. The challenges for control design in this class of vehicles lie in the inherent coupling between the propulsion system, and the airframe dynamics, and the presence of strong flexibility effects. Working from a highly nonlinear, dynamically-coupled simulation model, control designs are presented for velocity, angle-of-attack, and altitude command input tracking for a linearized version of a generic air-breathing hypersonic vehicle model linearized about a specific trim condition. Control inputs for this study include elevator deflection, total temperature change across the combustor, and the diffuser area ratio. Two control design methods are presented, both using linear quadratic techniques with integral augmentation, and are implemented in tracking control studies. The first approach focuses on setpoint tracking control, whereas in the second, a regulator design approach is taken. The effectiveness of each control design is demonstrated in simulation on the full nonlinear model of the generic vehicle.

Keywords: LQR, tracking, hypersonic, air-breathing, integral augmentation, control

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Nomenclature, States and Inputs

	States		Inputs
V_t	Vehicle velocity	δ_e	Control surface deflection
α	Angle-of-attack	ΔT_0	Total temperature change across combustor
Q	Pitch rate	A_d	Diffuser area ratio
h	Altitude	x_d	Cowl lip position
θ	Pitch angle		
η_i	Generalized elastic coordinates		
$\dot{\eta}_i$	Time derivatives of the elastic coordinates		

Table 1. Definitions of the states and inputs

I. Introduction

Air-breathing hypersonic vehicles may represent a more cost efficient way to launch small satellites or other vehicles into low earth orbit (LEO) than expendable rockets. With this type of aircraft, quick response and global strike capabilities for Air Force missions will be more practical. The recent success of NASA's X-43A scramjet powered airplane in flight testing has affirmed the feasibility of this technology. Still, much work remains before air-breathing technology can move from the drawing board to a permanent home in the skies.

Control of air-breathing hypersonic vehicles is a significant challenge because of the strong interaction between the aerodynamic and propulsive effects. As a matter of fact, the peculiar characteristic of air-breathing hypersonic vehicles is the strong interaction between the airframe and the engine dynamics.[1–3] Moreover, a substantial coupling exists between the engine thrust and the pitching moment. The vehicle's unique design results in non-conventional aircraft modes; the short period and phugoid are not defined in the traditional sense and the system response appears to be dominated by the altitude dynamics, which is only weakly controllable. The aircraft model is characterized by critically stable internal dynamics, and is statically unstable. Additionally, the body must be considered as a flexible structure due to its length, relative slenderness, and light weight. The elastic modes cause changes in the aerodynamic forces and moments of the vehicle. As a result, vehicles of this kind are notoriously difficult systems to control.

Due to the enormous complexity of the dynamics, only models of the longitudinal dynamics of air-breathing hypersonic vehicles have been developed and used for control design. There have been several attempts to address the challenges of model development and control of air-breathing hypersonic vehicles,[1,2,4–7] with a few concerted efforts to incorporate guidance and control.[8–10] In the cited references, linearized models of the vehicle dynamics have been considered for control design, with the noticeable exception of Tournes et al.,[9] where nonlinear variable structure control has been adopted. On the other hand, for hypersonic vehicle models with conventional actuation and negligible flexibility, the use of nonlinear control system design methodologies is more common.[11–14]

In this paper, we present some results on the development of linear controllers for the novel air-breathing hypersonic vehicle model developed by Bolender and Doman.[3] As opposite to the model developed by Chavez and Schmidt,[5,15] which resort to Newtonian impact theory, Bolender and Doman have employed compressible flow theory to determine the pressures and shock angles, resulting in a possibly more accurate and substantially more complex model. Figure 1 provides a schematic of the geometry for this generic vehicle. A Simulink version of the model developed by Bolender and Doman[3] has become an integral part of this study. The Simulink model incorporates the equations of motion and the engine/aerodynamics. With such a structure, one gains access to all the Simulink tools such as the numerical linearizing function `linmod`, used here to attain a linearized model of the vehicle around a certain trim condition. Because virtually the entire system is implemented in the Simulink block environment, most sub-calculations, such as pressures, are accessible directly from Simulink. Finally, it should be noted that Simulink provides an excellent platform for controller development and was therefore used for this study with the nonlinear model implemented for all simulations.

Since linear control techniques are employed in this preliminary study, a linearized model is obtained

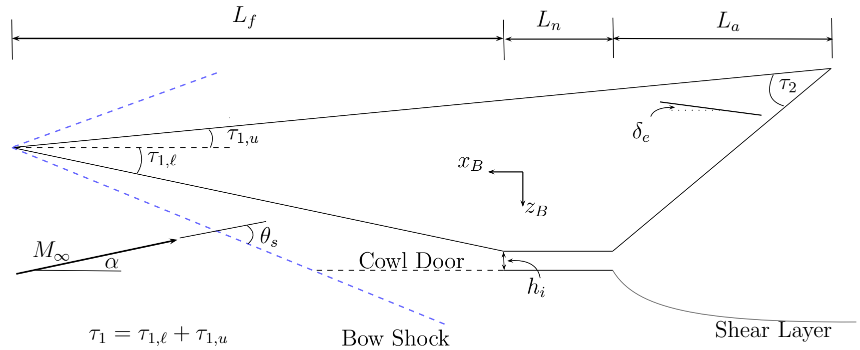


Figure 1. Vehicle Geometry for Hypersonic Air-breathing Vehicle[3]

for the nonlinear system at a specified trim condition, so that control can be designed for a region in a neighborhood of the operating point. The main goal of this study is to determine feasibility of linear control methods via designs on linearized models, for the generic nonlinear model. Extensions of the control system design would naturally involve gain scheduling around several trim conditions of interest. The trim condition used in this paper is given in Table 2. Controlled outputs for this problem are the angle-of-attack, the vehicle velocity, and the altitude of the vehicle. Since to achieve tracking it is necessary to have at least as many inputs as there are outputs to track,[16] a minimum of three control inputs is required in the case considered here. The control inputs are the control surface deflection (elevator), the total temperature change across the combustor, and the diffuser area ratio. The cowl door position x_d, is also available as a control input but it has kept fixed in this study, because early testing did not prove it as effective as the previously mentioned ones. The considered inputs provide control authority over the forces and moments associated with the nonlinear model. For example, the elevator deflection mainly affects the pitching moment as well as the lift and drag of the vehicle, though to a lesser extent it affects the other forces as well. Clearly, the temperature change across the combustor and the diffuser area ratio will mostly affect thrust.

In this work, two design methodologies are utilized for tracking control. For the first, an LQR controller is developed directly; for the second one, the tracking problem is cast into a regulation problem, to which LQR design can be applied. The linear quadratic regulator approach results in an optimal choice of system gains, accounting for the coupling inherent to the system, according to some cost criterion. While LQR design ensures a certain degree of robustness for the stability of the closed loop system in the presence of parametric uncertainties (even when applied to the nonlinear model), integral control is required to obtain zero steady state error with respect to constant setpoint tracking in the presence of model uncertainties. With integral action combined into the overall system, the feedback and feedforward gains for state feedback can be specified easily via LQR design. Similarly, an integrator can also be added to the regulator setup to achieve the same purpose.

The organization of this paper is as follows: In section II the system is defined with an outline of the control objectives and performance objectives. Then, in Section III a setpoint tracking controller is developed, followed by a regulator-type controller in Section IV. Simulation results on the nonlinear model are presented and discussed for both controllers in Section V. Finally, we draw some conclusions in Section VI.

II. Problem Formulation

The longitudinal dynamics for the vehicle model shown in Figure 1, as given in Bolender and Doman,[3] is described by the following nonlinear equations

$$\begin{aligned}
\dot{V}_t &= \frac{1}{m}\left(\mathcal{T}\cos\alpha - D\right) - g\sin(\theta-\alpha) \\
\dot{\alpha} &= \frac{1}{mV_t}\left(-\mathcal{T}\sin\alpha - L\right) + Q + \frac{g}{V_t}\cos(\theta-\alpha) \\
I_{yy}\dot{Q} &= M + \widetilde{\psi}_1\ddot{\eta}_1 + \widetilde{\psi}_2\ddot{\eta}_2 \\
\dot{h} &= V_t\sin(\theta-\alpha) \\
\dot{\theta} &= Q \\
k_1\ddot{\eta}_1 &= -2\zeta_1\omega_1\dot{\eta}_1 - \omega_1^2\eta_1 + N_1 - \widetilde{\psi}_1\frac{M}{I_{yy}} - \frac{\widetilde{\psi}_2\widetilde{\psi}_1\ddot{\eta}_1}{I_{yy}} \\
k_2\ddot{\eta}_2 &= -2\zeta_2\omega_2\dot{\eta}_2 - \omega_2^2\eta_2 + N_2 - \widetilde{\psi}_2\frac{M}{I_{yy}} - \frac{\widetilde{\psi}_1\widetilde{\psi}_2\ddot{\eta}_2}{I_{yy}},
\end{aligned}$$

where $\mathcal{T}, L, D, N_i$ are the thrust, lift, drag, and generalized elastic forces respectively, M is the pitching moment about the y-axis, I_{yy} is the moment of inertia, ζ_i, ω_i are the damping coefficients and natural frequencies of the elastic modes. Furthermore,

$$\begin{aligned}
k_1 &= 1 + \frac{\widetilde{\psi}_1}{I_{yy}} \\
k_2 &= 1 + \frac{\widetilde{\psi}_2}{I_{yy}} \\
\widetilde{\psi}_1 &= \int_{-L_f}^{0} \hat{m}_f\xi\phi_f(\xi)d\xi \\
\widetilde{\psi}_2 &= \int_{0}^{L_a} \hat{m}_a\xi\phi_a(\xi)d\xi,
\end{aligned}$$

where $\hat{m}_f$, $\hat{m}_a$ are the mass densities of the forebody and aftbody respectively and ϕ_f, ϕ_a are the mode shapes for the forebody and aftbody respectively. These equations assume a two cantilever model that is constrained at its origin so that the coupling cannot be ignored. Since only coupling between the pitch and flexible modes are considered in the flexible structure, the equations are more complex than if a free-free beam was used to approximate the structure. The states and inputs listed in Table 1 affect the forces and moment in these equations in a complex nonlinear way. The reader is referred to Bolender and Doman[3] for further details.

A linearized model about a trim condition of the nonlinear vehicle dynamics described by the equations above, was developed using numerical methods. The linearized equations read as

$$\begin{aligned}
\dot{x}_p &= A_p x_p + B_p u_p \\
y_p &= C_p x_p \\
z_p &= H_p x_p,
\end{aligned} \tag{1}$$

where $x_p \in \mathbb{R}^9$ is the state of the plant, $u_p \in \mathbb{R}^3$ is the control input, $y_p \in \mathbb{R}^9$ the output available for feedback, and $z_p = (V_t\ \alpha\ h)^T$ is the performance output to be regulated to a desired reference command. The state x_p, the input u_p, and the output z_p in (1) are all deviations of the corresponding trajectories of the non-linear system from the trim condition. The states and inputs are arranged in the same order as they are given in Table 1. The detailed expression of the plant matrices $A_p \in \mathbb{R}^{9\times 9}, B_p \in \mathbb{R}^{9\times 3}, C_p = I_{9\times 9}$ will not be explicitly given here for brevity. Since the only control inputs used in the case we consider here are δ_e, ΔT_0, and A_d, the fourth input x_d is fixed at its trim state and does not appear in the input vector. The linear controllers are designed on the basis of the linearized model, and their performance evaluated

by means of computer simulations on the full-scale nonlinear model. The desired outcome is to have the performance output of the system track a reference input, $r(t) \in \mathbb{R}^3$, which is assumed to have a constant steady state value.

For added model accuracy, a simple model of actuator dynamics has been added to the plant model. The actuator dynamics are modelled as

$$\dot{x}_\delta = A_\delta x_\delta + B_\delta u_\delta,$$

where

$$A_\delta = \begin{pmatrix} -20 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & -10 \end{pmatrix}, \quad B_\delta = \begin{pmatrix} 20 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{pmatrix},$$

$$x_\delta = \begin{pmatrix} x_{\delta_e} \\ x_{\Delta T_0} \\ x_{A_d} \end{pmatrix}, \quad u_\delta = \begin{pmatrix} u_{\delta_e} \\ u_{\Delta T_0} \\ u_{A_d} \end{pmatrix}.$$

The parameters of the actuator model are chosen to approximate values for real actuators. The actuator dynamics are appended to (1) as follows

$$\begin{aligned} \dot{x}_1 &= A_1 x_1 + B_1 u_1 \\ y_1 &= C_1 x_1 \\ z_1 &= H_1 x_1, \end{aligned} \tag{2}$$

where

$$A_1 = \begin{pmatrix} A_p & B_p \\ 0 & A_\delta \end{pmatrix}, \quad B_1 = \begin{pmatrix} 0 \\ B_\delta \end{pmatrix}, \quad H_1 = \begin{pmatrix} H_p & 0 \end{pmatrix},$$

$$C_1 = I_{12\times 12}, \quad x_1 = \begin{pmatrix} x_p \\ x_\delta \end{pmatrix}, \quad u_1 = u_\delta.$$

This allows the actuator states to be taken into account in the control system design. Therefore, the actuator bandwidth limitations are included in the controller design, but the presence of input saturations were not taken directly into account for this study. However, the results of closed loop simulations on the full nonlinear model show that reasonable responses can be generated well within the limitations that would be imposed on the actuators through a careful selection of the weighting matrices of the LQR design.

A. Control Objectives

The objective is to control the angle of attack, velocity, and altitude of the vehicle to a reference command representing a desired deviation from a given trim state, using as control inputs the control surface deflection, total temperature change across the combustor, and the diffuser area ratio. This will be done by designing full-state feedback controllers to achieve setpoint regulation in the neighborhood of a trim condition. This is obviously the first step towards the more ambitious goal of designing a gain scheduling controller by output feedback, since it may not be possible to measure all of the states of the system. Two different control methodologies will be employed:

- LQ design with implicit model following and integral augmentation.
- Regulator theory with model following and integral augmentation.

B. Performance Objectives

The final performance objective is to achieve a simultaneous setpoint tracking of the angle of attack (α), velocity (V_t), and altitude (h) without the actuators violating their position constraints. Control limitations were not included explicitly in the controller design but were considered in the tuning process. For the flight condition in this study the control limitations are as follows: the diffuser area ratio must satisfy $A_d \in (0,1)$, the temperature change ΔT_0 should be approximately less than 3000 deg R of the trim value, while the surface deflection must satisfy $\delta_e \in (-30°, 30°)$.

III. Setpoint Tracking Control

A. Integral Augmentation

It is desirable to include integral control into the state feedback to eliminate the steady state error. By augmenting the system with the integral error it is possible to let the LQR routine choose the value of the integral gain automatically. The advantage of adding the integrators is that it eliminates the need to determine the DC gain of the closed-loop system which would be difficult because of the uncertainty in the model. Let the error of the system (2) be given by the difference between the reference command $r(t)$ and the performance output $z(t)$, that is,

$$e_1 = r - H_1 x_1.$$

Then, let the integral error be given by,

$$x_2 = \int_0^t e_1(\tau) d\tau$$

and append the error dynamics to the system (2), to obtain

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} A_1 & 0 \\ -H_1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} B_1 \\ 0 \end{pmatrix} u + \begin{pmatrix} 0 \\ I \end{pmatrix} r.$$

Letting $x = \mathrm{col}(x_1, x_2)$, we write the above system as

$$\dot{x} = Ax + Bu + Gr, \tag{3}$$

with obvious meaning of the matrices A, B, and G. The new error for the system (3) can then be defined as

$$e = \begin{pmatrix} r - H_1 x_1 \\ x_2 \end{pmatrix} = Mr + Hx, \tag{4}$$

where

$$M = \begin{pmatrix} I \\ 0 \end{pmatrix} \quad H = \begin{pmatrix} -H_1 & 0 \\ 0 & I \end{pmatrix}.$$

B. LQ Optimal Control

The control design problem is to find a linear optimal tracking controller to minimize the cost function

$$J = \frac{1}{2} \int_0^\infty \left(e^T Q e + u^T R u \right) dt,$$

subject to (3), where $Q = Q^T \geq 0$, $R = R^T > 0$. By direct substitution we get,

$$J = \frac{1}{2} \int_0^\infty \left(x^T H^T Q H x + 2 r^T M^T Q H x + r^T M^T Q M r + u^T R u \right) dt. \tag{5}$$

Using the standard methodology, outlined for example in Bryson,[17] we arrive at the following differential equations

$$\dot{P} = -PA - A^T P - H^T Q H + P B R^{-1} B^T P \tag{6}$$
$$\dot{g} = (P B R^{-1} B^T - A^T) g - (H^T Q M + P G) r. \tag{7}$$

Equation (6) is the standard Riccati equation, while equation (7) is an auxiliary vector equation that defines the feedforward gain. It can be verified that (A, B) is stabilizable and $(A, \sqrt{Q})$ is detectable, therefore there exist a unique steady state solution ($\dot{P} = 0$) to equation (6), obtained by means of the associated algebraic Riccati equation (ARE),[18] whose solution will be denoted by P_{ss}. Assuming that we want to achieve perfect

tracking of $r(t)$ at its constant steady state value r_{ss}, then the following equation gives the steady state solution for g in the form

$$g_{ss} = (P_{ss}BR^{-1}B^T - A^T)^{-1}(H^TQM + P_{ss}G)r_{ss}.$$

We can now write the control law as

$$u = -K_x x - K_r r,$$

where

$$K_x = R^{-1}B^T P_{ss} \tag{8}$$
$$K_r = R^{-1}B^T(P_{ss}BR^{-1}B^T - A^T)^{-1}(H^TQM + P_{ss}G). \tag{9}$$

Finally, we can write the closed loop system as

$$\dot{x} = (A - BK_x)x + (G - BK_r)r.$$

Once the ARE is solved, then the optimal gains can be computed from (8) and (9). The problem now becomes how to choose Q and R so that a good response is obtained without exceeding the bandwidth and position limitations of the actuators. The position limits for the control effectors were imposed qualitatively, they were not explicitly included in the optimal control problem. It is worth noting that the gain computed in (8) and (9) are optimal only in steady state. In particular, the feedforward gain provides perfect tracking only when $r(t) = r_{ss}$.

C. Implicit Model Following (IMF) Method

It is very difficult to tune the weighting matrices to achieve an acceptable response for all the performance outputs while at the same time keeping the control effectors within their limits. In classical control, certain responses can be specified and sometimes achieved by deciding what damping ratio, overshoot, settling time, etc. are desired. While it is not straightforward to do so using LQR, it is possible to devise a methodology for picking the LQR weighting matrices in a simple and effective way. Using the implicit model following method, as described in Stevens and Lewis,[19] it is possible to make the selection of the weighting matrices more intuitive. This is done by first specifying a desired reference model for the error e in (4) as the solution of the autonomous differential equation

$$\dot{e}_m = A_m e_m,$$

where the matrix A_m specifies the desired dynamics for the error to follow. For instance, a desired convergence ratio for the error $e(t)$ can be selected a priori choosing the eigenvalues of A_m correspondingly, and the matrix A_m can be chosen either diagonal or block diagonal to enforce decoupling among selected components of $e(t)$. Define the model following error as

$$e_\rho = \dot{e} - A_m e, \tag{10}$$

where $\dot{e}$ is the derivative of the actual error. The model following error can then be used in a performance index as it is desirable to bring this error to zero. The following performance index penalizes the model following error and the inputs

$$J_\rho = \frac{1}{2}\int_0^\infty \left(e_\rho^T Q_m e_\rho + u^T R_m u\right) dt. \tag{11}$$

The tunable design parameters are $A_m \in \mathbb{R}^{6\times6}$, $Q_m \in \mathbb{R}^{6\times6}$, and $R_m \in \mathbb{R}^{3\times3}$. It is worth noting that although a selection of weighting matrices Q_m and R_m is still required in (11), this problem is considerably simpler than the original one. First, the dimension of Q_m is smaller than the dimension of Q, and the matrix A_m can be chosen diagonal as discussed above. Recalling that the reference input is constant at steady state (step input), we approximate the time derivative of the error $e(t)$ by

$$\dot{e} = H\dot{x}. \tag{12}$$

Substituting (4) and (12) into (10)

$$\begin{aligned} e_\rho &= H\dot{x} - A_m e \\ &= HAx + HBu + HGr - A_m Mr - AHx \\ &= (HA - A_m H)x + HBu + (HG - A_m M)r. \end{aligned}$$

In our case, it can be shown that $HB = 0$, thus the error takes the form

$$e_\rho = \bar{M}r + \bar{H}x, \tag{13}$$

where

$$\bar{M} = HG - A_m M, \quad \bar{H} = HA - A_m H.$$

Substituting (13) in equation (11) results in a performance index of the same form as equation (5) given in the previous section. For this case,

$$J = \frac{1}{2}\int_0^\infty \left(x^T \bar{H}^T Q_m \bar{H}x + 2r^T \bar{M}^T Q_m \bar{H}x + r^T \bar{M}^T Q_m \bar{M}r + u^T R_m u\right) dt.$$

Therefore, the derivation of the optimal gains given in the previous section still holds. The ARE resulting from (6), (8), and (9) can be solved using $\bar{M}$, $\bar{H}$, A, B, and G.

IV. Regulator Controller Design

A. Regulator

In this section, a second approach to the tracking problem will be pursued. It involves casting the tracking problem into a regulation problem and then designing a stabilizing controller using LQR techniques. The advantage to this approach lies in the fact that the feedforward control that is obtained guarantees regulation for a time varying reference, $r(t)$, as opposed to the previous method. Assume the reference is generated by the following autonomous system,

$$\begin{aligned} \dot{r}_f &= Sr_f \\ r &= C_f r_f, \end{aligned}$$

where $r_f \in \mathbb{R}^{n_f}$, $S \in \mathbb{R}^{n_f \times n_f}$, all the eigenvalues of the matrix S have non-positive real part. It can be verified that, for the linearized vehicle model under consideration and for the specific structure of the matrix S discussed in the next section, there exist matrices $\Pi \in \mathbb{R}^{12\times n_f}$, $\Gamma \in \mathbb{R}^{3\times n_f}$ that solve the Francis equation

$$\begin{aligned} \Pi S &= A_1\Pi + B_1\Gamma \\ 0 &= H_1\Pi - C_f, \end{aligned}$$

where A_1, B_1 and H_1 are described in (2). By inspection, the mappings $x_{ss} = \Pi r_f$ and $u_{ss} = \Gamma r_f$ define respectively the steady state trajectories for the state and the input of (2) which are compatible with the condition $e_1 = 0$.

Letting

$$\begin{aligned} \widetilde{x}_1 &= \Pi r_f - x_1 \\ \widetilde{u} &= \Gamma r_f - u, \end{aligned} \tag{14}$$

and substituting (14) into (2), an equation for the dynamics of $\widetilde{x}$ is derived, in the form

$$\begin{aligned} \dot{\widetilde{x}}_1 &= A_1\widetilde{x}_1 + B_1\widetilde{u} \\ e_1 &= H_1\widetilde{x}_1. \end{aligned} \tag{15}$$

The system (15) describes the dynamics of the tracking error. Note that (15) is independent of the actual reference inputs and that the tracking problem has been cast into a stabilization problem for the origin $\widetilde{x} = 0$ of (15). The Francis equation has a unique solution provided that the matrix[20,21]

$$\begin{pmatrix} A_1 - \lambda I & B_1 \\ -H_1 & 0 \end{pmatrix}$$

has full rank for any λ which is an eigenvalue of S. This condition, which states that no eigenvalue of S should be a transmission zero of the plant, can always be enforced by a proper selection of the matrix S. Furthermore, the integral of the error can be included to (15) by first defining $\dot{\widetilde{x}}_2 = -e_1$ and then appending $\widetilde{x}_2$ to the system

$$\begin{pmatrix} \dot{\widetilde{x}}_1 \\ \dot{\widetilde{x}}_2 \end{pmatrix} = \begin{pmatrix} A_1 & 0 \\ -H_1 & 0 \end{pmatrix} \begin{pmatrix} \widetilde{x}_1 \\ \widetilde{x}_2 \end{pmatrix} + \begin{pmatrix} B_1 \\ 0 \end{pmatrix} \widetilde{u}.$$

Letting $\widetilde{x} = \mathrm{col}(\widetilde{x}_1, \widetilde{x}_2)$ the equation above can be written as follows using the previously defined matrices

$$\begin{aligned} \dot{\widetilde{x}} &= A\widetilde{x} + B\widetilde{u} \\ e &= -H\widetilde{x}. \end{aligned} \tag{16}$$

The tracking problem has now been simplified into a standard regulator problem, that is, that of regulating $\widetilde{x}$ to 0.

B. IMF Applied to the Regulator

Consider again the system (16). We will apply the standard LQR, full state feedback control to this system. As in Section III, it is desirable to simplify the tuning process of choosing the weighting matrices Q and R by applying the implicit model following method. We start by defining the performance index (11), where e_ρ is defined in (10), and the error e is defined by the second equation in (16). After simple algebraic manipulations, we obtain the expression of the performance index as

$$J_\rho = \frac{1}{2} \int_0^\infty \left(\widetilde{x}^T Q_\rho \widetilde{x} + 2\widetilde{x}^T W_\rho u + u^T R_\rho u \right) dt,$$

where

$$\begin{aligned} Q_\rho &= (HA - A_m H)^T Q_m (HA - A_m H) \\ W_\rho &= (HA - A_m H)^T Q_m HB \\ R_\rho &= B^T H^T Q_m HB + R_m. \end{aligned}$$

Since in our case $HB = 0$, the above expressions are simplified into $W_\rho = 0$, $R_\rho = R_m$, and

$$J_\rho = \frac{1}{2} \int_0^\infty \left(\widetilde{x}^T Q_\rho \widetilde{x} + u^T R_\rho u \right) dt.$$

This problem can be solved using a conventional LQR algorithm provided that $Q_\rho = Q_\rho^T \geq 0$, $R_\rho = R_\rho^T > 0$, (A, B) is stabilizable and that $(A, \sqrt{Q_\rho}\ )$ is detectable. The tunable design parameters are $A_m \in \mathbb{R}^{6\times 6}$, $Q_m \in \mathbb{R}^{6\times 6}$, $R_\rho \in \mathbb{R}^{3\times 3}$, and $S \in \mathbb{R}^{n_f \times n_f}$ which defines the exosystem. We assume that S is tunable because in our case it is the state space realization of a step reference and a tunable second order command shaping filter. The solution will be of the form

$$\widetilde{u} = -K_\rho \widetilde{x},$$

which as expected, does not contain a feedforward component per se, since we are merely implementing state feedback on system (16). However, after substituting with (14) we see that the actual control input can be written as a combination of feedback and feedforward terms as follows

$$u = \Gamma r + K_\rho \widetilde{x} = \Gamma r - K_\rho \begin{pmatrix} x_1 - \Pi r \\ -\widetilde{x}_2 \end{pmatrix}.$$

V. Simulation Results for the Nonlinear System

To test the effectiveness of each controller developed in this study, its performance has been evaluated in simulation on the Simulink model discussed in the introduction. To implement a linear controller on a nonlinear model the trim state values must be subtracted from the states of the nonlinear model and the inputs are fed by the outputs of the linear controller with the trim condition added.

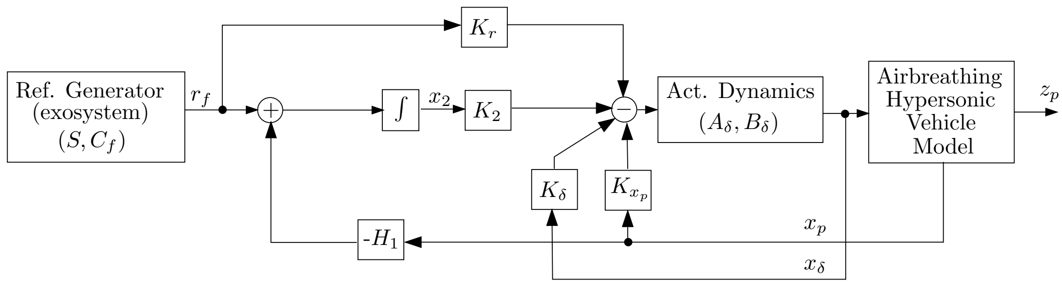


Figure 2. Block Diagram of Setpoint Tracking Controller

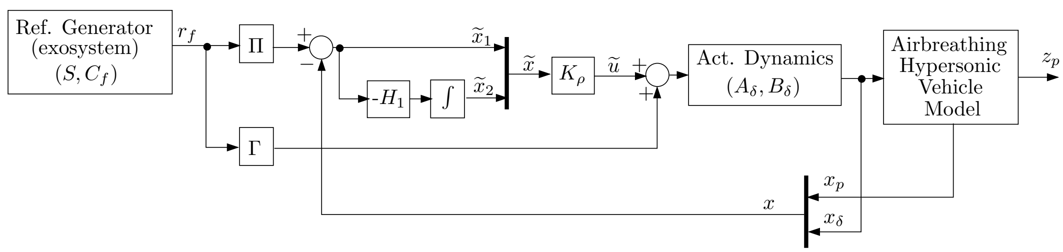


Figure 3. Block Diagram of Regulator Controller

A. Filtering the Reference Input

The components of the input reference vector have been chosen as step inputs, filtered by three decoupled 2^{nd} order command shaping filters of the following form

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2},$$

where ζ is the damping coefficient and ω_n is the natural frequency of the filter. While the filter dynamics were not included in the design of the setpoint tracking controller, they were explicitly incorporated into the regulator design. By writing the command reference filters in state space, a state space representation of the reference generator as an exosystem can be formulated as follows

$$\begin{aligned} S &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & I_{3\times 3} \\ \Omega_n^2 & -\Omega_n^2 & -2\Xi\Omega_n \end{pmatrix} \\ C_f &= \begin{pmatrix} 0_{3\times 3} & I_{3\times 3} & 0_{3\times 3} \end{pmatrix}, \end{aligned}$$

where Ω_n and Ξ are real 3×3 diagonal matrices that define the natural frequencies and damping ratios of the command reference filters respectively (see the Appendix), and the initial value of the first three states define the amplitude of the reference command in steady state. A block diagrams for each controller is given in Figure 2 and Figure 3 respectively.

B. Simulation Parameters and Results

The performance objective is to achieve a simultaneous set point tracking of the following steady state deviations from the trim state: 1° for α, 1000 ft/s for V_t, and 10000 ft for h, which are reasonable requirements for a vehicle of this type. The steady state demand on the outputs place a physical restriction on the transient response of the vehicle. For example, it would be physically impossible for the aircraft to gain 10000 ft in altitude over a short time interval. Therefore, the criteria of the transient response, such as rise time, settling time, and overshoot, were determined a priori to provide reasonable physical results. Using simple G-force

calculations it was determined that a desirable response should have a settling time of approximately 1.3 sec for α, 102 sec for V_T, and 60 sec for h with as little overshoot as reasonably possible for the given step sizes. This criteria proved too difficult to be satisfied without exceeding the physical limitations for the actuators. If an anti-windup scheme were added to the controllers this might be possible but no input saturation was considered in the design of the controllers in this study. As previously discussed, these limitations were only taken into consideration during the tuning process, it was therefore decided to relax the criteria by tuning the exosystem, i.e. the step reference input filter. The results in Figure 4, show that both controllers produce inputs within the given limitations and give a slightly faster response then the filtered reference without considerable overshoot. The settling times of the reference inputs are as follows: 3.6 sec for α, 166 sec for V_T, and 117 sec for h. These are very reasonable results considering that we are not using saturations in our controllers, as well as the fact that we are using a linearized model for the controller design and are moving considerably away from the trim condition. The trim condition is given in Table 2.

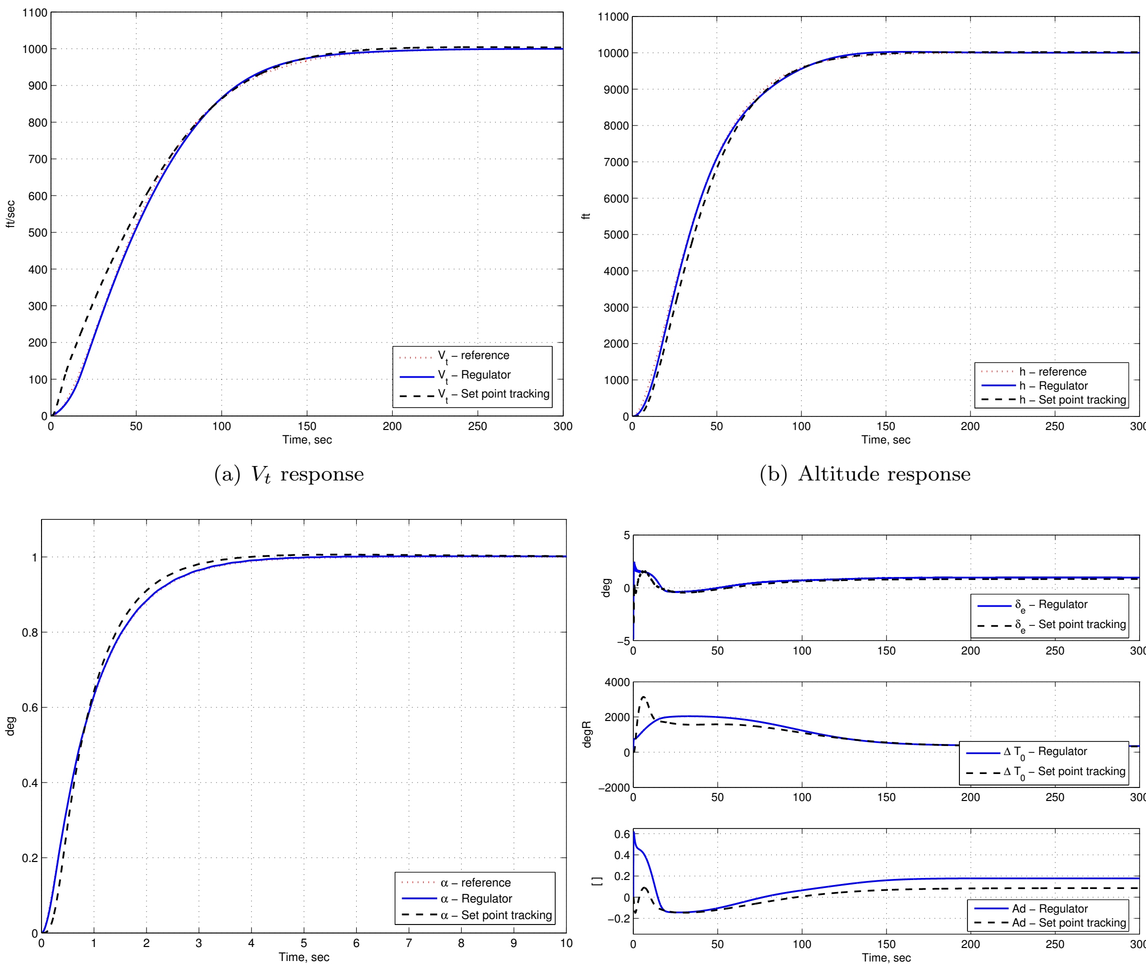
States:		Inputs:	
V_t [ft/s]	7846.4	δ [rad]	0.13716
α [rad]	0.034217	ΔT_0 [deg R]	484.49
Q [rad/s]	0	A_d	0.35005
h [ft]	85000	x_d [ft]	7.0967
θ [rad]	0.034217		
η_1	1.6105		
$\dot{\eta}_1$	0		
η_2	1.4582		
$\dot{\eta}_1$	0		

Table 2. Trim condition

It should be kept in mind that tuning the weighting matrices of the LQR problem for both controllers can vastly alter the resulting performance. For both control methods, it was possible to obtain a selection of the weighting matrices that produced satisfactory performance with respect to the outlined design criteria (see Appendix), which is clearly illustrated in Figure 4. The results shown in this figure describe several differences between the two control approaches. The regulator approach incorporates the reference filter in the controller design, where the setpoint tracking approach does not. This allows the regulator approach to achieve a much tighter tracking of the reference input during transient. This effect can be seen in the simulation results. The transient behavior of the actuators differ between the two control methods. This results from a difference in the way the feedforward gain is calculated for each method. Despite these differences, both controllers provide asymptotic tracking with no significant overshoot or steady state error.

VI. Conclusions

For a nonlinear model of a generic air-breathing hypersonic vehicle, it is a difficult task to achieve tracking with zero steady state error, and still retain favorable responses when commanding V_t, α, and h at the same time. The results demonstrated in this paper show that satisfactory responses are possible with linear controllers in a neighborhood of a trim condition. The design methodology proposed in this paper reposes upon variations of LQR design with integral augmentation. While bandwidth or amplitude limitations are not explicitly accounted for in the controller design, it is possible to tune the controller parameters so that the inputs remain feasible. Further investigation is needed to determine the region of validity of the linear controllers when applied to the nonlinear model, although their effectiveness is guaranteed at least in a neighborhood of the trim condition. A study of this kind is needed to implement a gain scheduling control scheme that encompasses the entire flight envelope. Future work will address more directly the issues of hard constraints on the control inputs, as well as output feedback design.



(a) V_t response

(b) Altitude response

(c) Angle-of-attack response

(d) Control Inputs

Figure 4. Performance outputs and control inputs to the non-linear plant shown as deviations from trim condition applying the two control methods separately.

VII. Acknowledgements

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Appendix: Tuning Parameters and Computed Gains

A. Set Point Tracking Control Approach

$$\begin{aligned}
\Omega_n &= \mathrm{diag}(\ 3 \quad 0.035 \quad 0.05\ ) \\
\Xi &= \mathrm{diag}(\ 1.5 \quad 1 \quad 1\ ) \\
A_m &= \mathrm{diag}(\ -300 \quad -3.5 \quad -5 \quad -3 \quad -0.035 \quad -0.05\ ) \\
Q_m &= \mathrm{diag}(\ 1e+005 \quad 10 \quad 10 \quad 1e+010 \quad 10 \quad 1\ ) \\
R_m &= \mathrm{diag}(\ 1e+006 \quad 0.05 \quad 1e+007\ )
\end{aligned}$$

$$K_x = \begin{pmatrix}
\text{-3.84e-005} & \text{-156} & \text{-21.4} & \text{-0.00145} & \text{-22.1} & 0.0366 & 0.00294 & \dots \\
42 & \text{-8.96e+005} & \text{9.83e+003} & 43.9 & \text{8.88e+005} & 1.83 & -1.55 & \dots \\
-0.00223 & -108 & 1.75 & 0.00392 & 109 & -0.000221 & -0.000273 & \dots
\end{pmatrix}$$

$$\begin{pmatrix}
\dots & -0.00162 & -0.000136 & 1 & \text{5.31e-006} & -0.109 & 299 & \text{3.24e-007} & \text{4.53e-006} \\
\dots & -0.115 & -0.115 & 53.1 & 0.0569 & 313 & \text{7.82e+004} & -0.389 & -0.138 \\
\dots & \text{1.01e-006} & \text{-1.63e-005} & -0.00543 & \text{1.56e-006} & 0.0457 & 6.58 & \text{2.16e-005} & \text{-1.24e-005}
\end{pmatrix}$$

$$K_r = \begin{pmatrix}
178 & 4.19e-005 & 0.00144 \\
2.61e+004 & -41.8 & -44 \\
18.3 & 0.00235 & -0.00397
\end{pmatrix}$$

B. Regulator Approach

$$\begin{aligned}
\Omega_n &= \mathrm{diag}(\ 3 \quad 0.035 \quad 0.05\ ) \\
\Xi &= \mathrm{diag}(\ 1.5 \quad 1 \quad 1\ ) \\
A_m &= \mathrm{diag}(\ -60 \quad -0.35 \quad -0.5 \quad -930 \quad -0.35 \quad -0.5\ ) \\
Q_m &= \mathrm{diag}(\ 100 \quad 4 \quad 0.0001 \quad 2e+003 \quad 8 \quad 0.005\ ) \\
R_m &= \mathrm{diag}(\ 200 \quad 2 \quad 2e+005\ )
\end{aligned}$$

$$\Pi = \begin{pmatrix}
0 & 0 & 0 & 0 & 1 & 0 & \text{3.36e-015} & 0 & 0 \\
\text{-2.13e-015} & \text{8.38e-022} & \text{-2.03e-021} & 1 & \text{-1.57e-016} & \text{-1.93e-018} & \text{-2.06e-015} & \text{-7.49e-018} & \text{-5.66e-020} \\
\text{-2.34e-014} & \text{-9.18e-021} & \text{3.19e-007} & \text{3.91e-014} & \text{-1.65e-017} & \text{-3.19e-007} & 1 & \text{-1.55e-016} & \text{-1.27e-005} \\
0 & 0 & 0 & 0 & 0 & 1 & \text{4.66e-013} & 0 & 0 \\
\text{-1.6e-015} & \text{8.38e-022} & \text{-2.03e-021} & 1 & \text{-1.57e-016} & \text{-1.93e-018} & \text{-2.6e-015} & \text{-7.49e-018} & 0.000127 \\
-4.34 & \text{-1.35e-009} & \text{1.68e-008} & 29.9 & 0.000304 & \text{-7.29e-005} & 4.42 & \text{-6.45e-007} & \text{-3.39e-007} \\
39.7 & \text{-7.9e-010} & \text{-8.47e-010} & -39.7 & \text{7.9e-010} & \text{8.47e-010} & -9.88 & 0.000304 & \text{-7.28e-005} \\
-60 & \text{-4.9e-008} & \text{3.15e-005} & 76.6 & 0.000194 & \text{-7.82e-005} & 61.1 & 0.0214 & -0.00117 \\
550 & \text{2.62e-005} & \text{-2.92e-006} & -550 & \text{-2.62e-005} & \text{2.92e-006} & -474 & -0.0013 & \text{3.87e-005} \\
-3.66 & \text{-2.17e-010} & \text{7.44e-006} & 2.3 & \text{-1.31e-005} & \text{-3.01e-006} & 3.66 & -0.00132 & -0.000303 \\
\text{-1.09e+004} & \text{3.34e-007} & 0.0746 & -60.1 & -0.117 & -0.0268 & \text{1.09e+004} & 140 & -2.41 \\
32.5 & \text{-1.76e-010} & \text{7.2e-005} & -51 & -0.000116 & \text{-2.97e-005} & -32.5 & -0.00997 & -0.00292
\end{pmatrix}$$

$$\Gamma = \begin{pmatrix}
-2.01 & \text{-8.08e-008} & \text{7.4e-006} & 0.649 & \text{-1.3e-005} & \text{-2.97e-006} & 2.13 & -0.00131 & -0.000302 \\
\text{-1.11e+003} & 0.0171 & 0.074 & \text{-9.84e+003} & -0.134 & -0.0262 & \text{1.08e+003} & 139 & -2.39 \\
3.29 & \text{-1.22e-006} & \text{7.12e-005} & -21.8 & -0.000114 & \text{-2.89e-005} & -8.35 & -0.00991 & -0.00289
\end{pmatrix}$$

$K_\rho =$

$$\left(\begin{array}{cccccccc} \text{-0.0238} & \text{-585} & \text{-78.5} & \text{-0.00194} & \text{-94.2} & 0.111 & 0.0111 & \ldots \\ 13.9 & \text{-2.29e+004} & \text{-127} & 0.197 & \text{2.29e+004} & 0.00985 & 0.026 & \ldots \\ \text{-0.00489} & \text{-61.7} & 1.08 & 0.00106 & 62 & \text{-6.4e-005} & \text{-0.00017} & \ldots \end{array} \right.$$
$$\left. \begin{array}{cccccccccc} \ldots & \text{-0.0288} & 0.000648 & 6.18 & \text{-0.000159} & \text{-0.361} & \text{-2.94e+003} & \text{-0.00126} & \text{-0.000152} \\ \ldots & 0.00149 & 0.00266 & \text{-0.00796} & 0.00991 & \text{-0.179} & \text{-1.1e+003} & 0.659 & 0.00843 \\ \ldots & \text{-3.17e-006} & \text{-1.16e-005} & \text{-0.000181} & \text{-1.79e-006} & 0.0288 & \text{-4.74} & \text{-0.00075} & \text{7.43e-005} \end{array} \right)$$

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